Surname .... **Candidate number** ....

First name ....

Current school ....

The Manchester Grammar School

Founded 1515

Entrance Examination 2016
Arithmetic Section B

1 Hour

Do not open this booklet until told to do so

Calculators may not be used

Write your names, school and candidate number in the spaces provided at the top of this page.

For each question, show all your working in full, as this will be marked, and then write your answer clearly in the space provided.

You have 1 hour for this paper which is worth 80 marks.

Marker	Short Problems Q1 - 6	Longer Problems Q7 - 11	**TOTAL**
Score			
out of	**30**	**50**	**80**

1. Complete this bill for a small shopping trip, filling in the **five** missing quantities and amounts in the spaces provided

	£. p
.... biscuits at 45p each	3.60
.... eggs which cost £1.60 for twelve	0.80
.... grams of butter at £2.50 per Kg	1.50
.... litres of milk costing 90p per litre	....
TOTAL	£ 8.15

[5 marks]

2. (a) Martin was born on 13th August 2000. How many birthdays did he have between 1st August 2001 and 1st September 2015?

2a	

(b) Paul was born on 10th November 2002. How many birthdays did he have between 20th November 2005 and 1st November 2015?

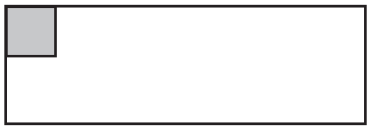
2b	

(c) Andy was born on 29th February 2004, which was a leap year. How many true birthdays could he celebrate between 1st January 2005 and 31st December 2015?

2c	

[4 marks]

3. A group of children are cutting squares off one corner of rectangular sheets of paper, as shown in the diagram.



(a) Ahmed's sheet of paper is 8 cm by 7 cm. He cuts out a square with sides of length 5 cm. What area of paper is remaining when he has cut out his square?

3a cm^2

(b) Bella's sheet of paper is 11 cm by 12 cm. After her square is cut out, the area of paper she is left with is 68 cm^2. What is the length of each side of the square she cuts out?

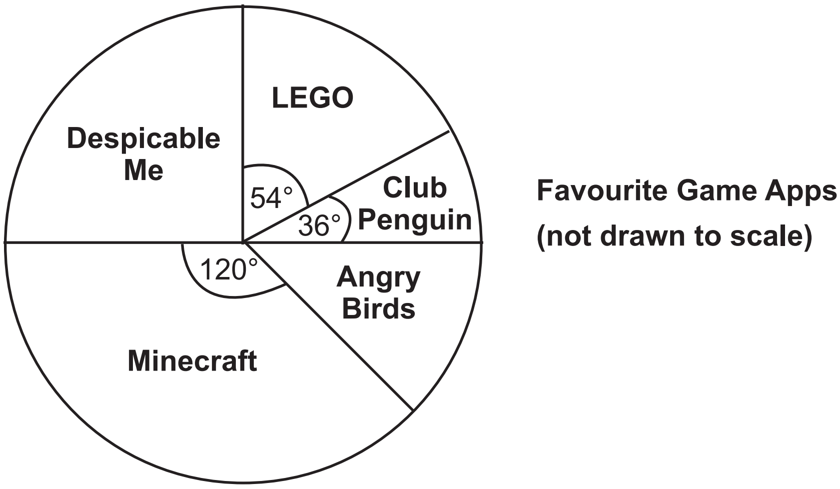
3b cm

(c) Chris has an area of 23 cm^2 of paper left when he cuts a square with sides of 7 cm from his sheet of paper. If his rectangular sheet of paper is 8 cm wide, how long is it?

3c cm

[5 marks]

4. Sixty pupils each voted for their favourite game app. The pie chart below shows how they voted



(a) What fraction of the class voted for Minecraft?

4a	

(b) One quarter of the pupils voted for Despicable Me.
What angle in the pie chart represents Despicable Me?

4b	°

(c) How many pupils voted for Angry Birds?

4c	

[5 marks]

5. In the MGS running competition, runners are placed in five heats and their time and position in their heat is used to work out when they can start in the final 'Rusholme Rally' race.

The results in the heats were as follows, the times are all in minutes and seconds

	Heat 1		Heat 2		Heat 3		Heat 4		Heat 5	
Position	Runner	Time	Runner	Time	Runner	Time	Runner	Time	Runner	Time
1st	A	1m 45s	D	1m 30s	G	2m 05s	J	1m 40s	M	1m 35s
2nd	B	2m 03s	E	1m 58s	H	2m 20s	K	1m 50s	N	1m 55s
3rd	C	2m 30s	F	2m 25s	I	2m 50s	L	1m 59s	O	2m 40s

In the final 'Rusholme Rally' race, the winner of each heat is given a 20 second handicap, the second place runner is given a 10 second handicap and any runner with a time faster than two minutes is given an **extra** 5 second handicap. This means that in the final 'Rusholme Rally' race, runners with no handicap set off when the start is signalled. Any runner with a 5 second handicap sets off 5 seconds after the start and similarly for the other time handicaps.

(a) List all the runners who set off when the start of the 'Rusholme Rally' is signalled because they have no handicap.

5a	

(b) Which runner has a 20 second handicap in the 'Rusholme Rally'?

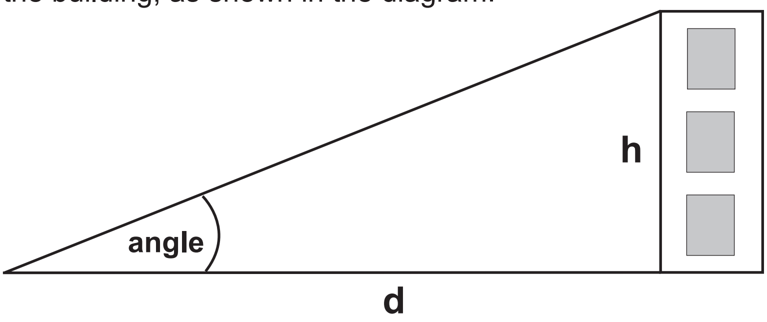
5b	

(c) Which runners have a handicap of 15 seconds?

5c	

[5 marks]

6. Howard discovers a method to find the heights of buildings. He measures the distance to the foot of the building, **d metres.** Then he measures the angle to the horizontal when he looks up at the top of the building, as shown in the diagram.



Using that angle, he then finds the quantity called the **tannangle** from the table below

Angle °	10	20	30	40	50	60	70	80
Tannangle	0.2	0.4	0.6	0.8	1.2	1.7	2.7	5.7

The height of the building, **h metres,** is given by the following calculation

$$h = d \times \text{tannangle}$$

e.g. if the building is 50 m away and the angle is 30° then the height is given by

$$h = 50 \times 0.6 = 30 \text{ m}$$

(a) Find the height of a building 20 m away when the angle is 60°.

6a	m

(b) Find the distance to a building 24 m high when the angle is 40°.

6b	m

(c) Find the angle if a building 100 m away is 270 m high.

6c	°

[6 marks]

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Short problems	**/30**

7. Groups of car enthusiasts are going to a car festival. To get to the place where the festival is happening they have a number of different sizes of car available as follows

Two seater sports cars, **Four** seater cars, **Six** seater people carriers

In order to keep the cost down, **each vehicle used on the journey is always full.**

(a) The first group use four sports cars, 6 four seater cars and two people carriers. How many are there in the group?

7a	

(b) The second group has 76 people in it. How many four seater cars will they need if they take 5 sports cars and 7 people carriers?

7b	

(c) The third group has 112 people in it. They use 8 four seater cars and equal numbers of sports cars and people carriers. How many of each do they need?

7c	

(d) In the fourth group there are 66 people. They need twice as many sports cars as four seater cars and twice as many four seater cars as people carriers. How many four seater cars do they use?

7d	

[8 marks]

8. This question is about the Recs of numbers - you are **NOT** expected to know about Recs.
The method for working out the Rec of two numbers is as follows

$$\textbf{Rec}\ (a,b) = \frac{a + b}{a \times b}$$

so

$$\textbf{Rec}\ (2,3) = \frac{2 + 3}{2 \times 3} = \frac{5}{6}$$

and where possible, the fraction answer is simplified

so

$$\textbf{Rec}\ (2,4) = \frac{2 + 4}{2 \times 4} = \frac{6}{8} = \frac{3}{4}$$

Using this method

(a) Work out Rec (4,5)

8a	

(b) Work out Rec (30,50)

8b	

(c) If Rec (a,a) = 1, find the value of a.

8c	

(d) Work out (i) Rec (3,3)

8di

(ii) Rec (5,5)

8dii

(iii) Rec (11,11)

8diii

(e) What do you notice about your answers in part d?

8e

[10 marks]

9. The stopping distance for a car is made up of two parts. The first is the distance travelled by the car while the driver is reacting to something that makes them want to brake. This is called the **Thinking Distance.** The second is the distance travelled by the car while the brakes are applied. This is called the **Braking Distance.** The **Stopping Distance** is given by adding these two distances together.

So **Stopping Distance = Thinking Distance + Braking Distance**

The table below shows the **Thinking Distance,** in metres, for various speeds in km per hour.

Speed (kmph)	Thinking Distance (m)
40	12
50	15
60	18
70	21
80	24
90	27

The **Braking Distance** in metres is given by the formula

$$\textbf{Braking Distance} = \frac{1}{60} \times \textbf{speed} \times \textbf{speed} \ \textbf{or} = \frac{1}{60} \times (\textbf{speed})^2$$

(a) If the Thinking Distance is 24 m, what speed was the car travelling at?

9a	kmph

(b) What is the Braking Distance of a car travelling at 90 kmph?

9b	m

(c) What is the Stopping Distance of a car travelling at 90 kmph?

9c	m

(d) A driver sees a child start to cross the road 80 m in front of his car. What distance would the car be from the child when the driver stopped if he was initially travelling at 60 kmph.

9d	m

[10 marks]

10. On January 1st 2013 a new spymaster recruits 4 new spies. On January 1st every following year he recruits twice as many new spies as he did the previous year.

Exactly two years after being recruited each spy recruits two new spies and each year after that recruits twice as many as the year before, so the four spies recruited by the spymaster in 2013 would recruit a total of eight new spies in 2015 as shown in the second column of the table. These eight spies would then recruit 16 new spies in 2016. Also, the eight spies recruited by the spymaster in 2014 would recruit 16 new spies in 2016 which is why the entry in the second column for 2016 is a **total** of 32.

Complete the table of spies recruited by the spymaster and his spies up to 2018, putting an answer on the dotted line in each of the spaces below.

	New spies recruited that year by spymaster	New spies recruited that year by other spies	TOTAL number of new spies recruited that year	TOTAL number of spies recruited since 2013
2013	4	0	4	4
2014	8	0	8	12
2015	16	8	....	....
2016	....	32	....	100
2017	64	....	....	260
2018	....	256	....	644

[10 marks]

11. The output of a heater is measured in watts and kilowatts. 1 kilowatt (kW) is equal to 1000 Watts so, for example a 2.6 kW heater produces 2600 Watts.

The Retention Factor (RF) of an insulating layer (which is material that stops heat flowing out) shows what percentage of the heat that reaches the insulating layer is kept in, and also allows you to work out what percentage of the heat escapes from the other side.

The table below gives you some examples of the percentages for certain RFs. Use the patterns in the table to work out the percentages you will need in the questions.

Retention Factor (RF)	Percentage of heat kept in	Percentage of heat that escapes
100	100	0
60	60	40
20	20	80
10	10	90

This means that, for example, with a 5kW heater, the number of Watts escaping through a 60RF layer is given by

$$\mathbf{5000 \times 40 \div 100 = 2000\ Watts = 2kW}$$

Using this method, work out the answers to the following questions

(a) With a 3 kW heater, how many Watts of heat escape through a 70RF insulating layer?

11a	Watts

(b) With a 2 kW heater, how much heat escapes through two 90RF layers put side by side?

11b	Watts

(c) With a 1 kW heater, how many 80RF layers are needed so that the heat that escapes is less than 5 Watts?

11c	

(d) A 4 kW heater is in front of three layers with a 50RF layer nearest the heater then a 40RF layer then a 30RF layer. What heat **escapes** between the second (40RF) and third (30RF) layers?

11d	Watts

(e) From a 4 kW heater only 108 Watts escape through three **identical** layers. What is the RF of each layer?

11e	RF

[12 marks]

This is the end of the Examination

Use any remaining time to check your work or try any questions you have not answered.

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Long problems	/50